Oberseminar
Basics on Automorphic Representations

WS 22/23

In this seminar we want to learn some basics about automorphic representations and automorphic forms. We will closely follow the recent book [GH22] (see also the lecture notes [Get]), however the speakers are encouraged to add examples and explanations. In the talks the focus should be on clear statements and examples rather than explaining details of the proofs. Notions from the theory of linear algebraic groups should be recalled as needed.

Below you find the list of suggested talks, which essentially follows the suggested path for an introductory course in the preface of [GH22]. If there is no explicit reference written, the numbering refers to [GH22].

1. Adeles (26.10.22)

2.1 - 2.3 (You can be a bit rough in 2.1 and more detailed in 2.2 and 2.3)

Lukas

2. The Haar measure (02.11.22)

3.2, 3.5, and 3.6.

Oliver

3. The Adelic quotient (09.11.22)

2.6, in particular explain the application of Thm 2.6.1 to the finiteness of the class number of a global field and, in case F is a function field of a smooth projective curve C over a finite field, the interpretation of $\mathrm{GL}_n(F)\backslash\mathrm{GL}_n(\mathbb{A}_F)/\mathrm{GL}_n(\widehat{\mathcal{O}}_F)$ in terms of isomorphism classes of rank n vector bundles on C, see e.g. [Gai03, 2.1] (If you like, you can also add that for G commutative, one has $G(F)\backslash G(\mathbb{A}_F)/G(\widehat{\mathcal{O}}_F) = H^1(C, G)$.) Discuss the homeomorphism (2.20) and the example for $\mathrm{GL}_{2,\mathbb{Q}}$ at the end.

Mattia

4. Automorphic Representations in the L^2-sense (16.11.22)

In this talk we see a first approximation of automorphic represention (Def 3.3). To this end explain what is needed from 3.1, 3.3, 3.4 and 3.7. (If you have time left you can explain, e.g. 3.8).

Sonia

5. Smooth vectors and representations of compact groups (23.11.22)

4.2 and 4.3.

Dennis

6. $(\mathfrak{g}, K)$-modules, infinitesimal characters, and classification of $(\mathfrak{g}, K)$-modules for $\mathrm{GL}_{2\mathbb{R}}$ (30.11.22)

4.4, 4.6 and 4.7.

Jens E.

7. Smooth and admissble representations (07.12.22)

5.1 - 5.3.

Sascha

8. Unramified Hecke-Algebra and Flath's Theorem (14.12.22)

5.5 - 5.7. Explain first what is needed from §2.4 (in particular Def 2.8 of hyperspecial subgroups).

Kay

9. Automorphic forms and - representations (21.12.22)

6.1 - 6.3 (with more details for 6.3). In case there is time left explain 6.4 (= automorphic representation in the function field case).

Fei

10. Cuspidal automorphic representations and modular forms (11.01.23)

6.5 and 6.7. If there is time left you could explain 6.8 or Thm 6.6.4 a bit.

Matthias

11. Unramified representations, the Satake Isomorphism, and the Langlands dual group (18.01.23)

7.1 - 7.3.

Britta

12. Satake Isomorphism for unramified groups and principal series (25.01.23)

7.5 (State Thm 7.1 and Cor 7.5.2 and explain the map $\mathcal{S}$ in (7.33).) Explain 7.6.

Georg

13. ....? (01.02.23)

N.N.

References

[Gai03] D. Gaitsgory. Informal introduction to geometric Langlands. In *An introduction to the Langlands program (Jerusalem, 2001)*, pages 269–281. Birkhäuser Boston, Boston, MA, 2003.

[Get] Jayce R. Getz. An introduction to automorphic representations. Lecture notes, `https://services.math.duke.edu/~jgetz/aut_reps`.

[GH22] Jayce R. Getz and Heekyoung Hahn. *An introduction to Automorphic Representations, with a view toward Trace Formulae*. 2022. `https://sites.duke.edu/jgetz/files/2022/04/Graduate_Text.pdf`.