Local–Global Diagrams for G–Equivariant $\widehat{\mathcal{D}}_\Omega$–Modules on the Drinfeld Plane

Andrés Sarrazola–Alzate*

(Ongoing joint work with Tobias Schmidt.)

Abstract

Let F be a non–Archimedean local field and set $G = \mathrm{GL}_2(F)$. We develop a local–global, fully diagrammatic description of G–equivariant, coadmissible $\widehat{\mathcal{D}}_\Omega$–modules on Drinfeld's upper half–plane Ω. The geometric input is the reduction map $\rho : \Omega \to \mathcal{T}$ to the Bruhat–Tits tree of $\mathrm{PGL}_2(F)$. For each vertex $x \in \mathcal{T}$ and (closed) edge $e \in \mathcal{T}$ we consider the affinoids $\Omega_x := \rho^{-1}(x)$ and $\Omega_e := \rho^{-1}(e)$. We show that the G–stable cover $\{\Omega_x, \Omega_e\}$ is Leray for coadmissible $\widehat{\mathcal{D}}$–modules, so global sections are computed by a small Čech equalizer involving only vertex and edge restrictions.

Fixing a vertex–edge pair (x_0, e_0), we attach to any $M \in \text{Coadm-}\widehat{\mathcal{D}}_\Omega^G$ its *local–global diagram* consisting of the local pieces $M_x := M(\Omega_x)$ and $M_e := M(\Omega_e)$ (with actions of $K_x = \mathrm{Stab}_G(x)$ and $I_e = \mathrm{Stab}_G(e)$) together with the restriction morphism $M_e \to M_x$ whenever x lies on e. Our main theorem is an equivalence of abelian categories

$$\text{Coadm-}\widehat{\mathcal{D}}_\Omega^G \;\simeq\; \mathrm{Diag}(x_0, e_0),$$

so every global G–equivariant coadmissible $\widehat{\mathcal{D}}_\Omega$–module is determined, up to unique isomorphism, by the diagrammatic data on a single vertex and an adjacent edge. As a consequence, simplicity, Jordan–Hölder series, and all extension groups Ext^i are detected purely diagrammatically.

We discuss applications: (i) a geometric irreducibility criterion for generic locally analytic principal series via diagrammatic simplicity; (ii) automatic diagrammatic simplicity for G_0–equivariant line bundles with nontrivial integrable connection; and (iii) geometric realizations of trianguline filtrations through Ardakov–Wadsley localization and vertex–edge restriction data. Time permitting, we also indicate the rank–one classification in terms of the equivariant Picard group $\mathrm{Pic}^G(\Omega)$ and the obstruction map $d\log : \mathrm{Pic}^G(\Omega) \to H^1(G, \Omega^1(\Omega))$.

*Universidad EIA.